Class XII Session 2025-26
Subject - Applied Mathematics
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study-based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If AB = A and BA = B, then $(B^2 + B)$ equals: **[1]**

a) 2A b) 2I

c) O d) 2B

2. Standard deviation of a sample from a population is called a **[1]**

a) Statistic b) Standard error

c) Parameter d) Central limit

3. A certain sum of money amounts to ₹ 5832 in 2 years at 8% p.a. compound interest. The sum invested is **[1]**

a) ₹ 5000 b) ₹ 5200

c) ₹ 5280 d) ₹ 5400

4. Comer points of the feasible region for an LPP are : (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let z = 4x + 6y be the objective function. Then, Max. z - Min. z = **[1]**

a) 42 b) 48

c) 18 d) 60

5. If B is a non-singular matrix and A is a square matrix, then det $(B^{-1} AB)$ is equal to **[1]**

a) Det (B^{-1})
b) Det (A^{-1})
c) Det (B)
d) Det (A)

6. If the mean and standard deviation of a binomial distribution are 12 and 2 respectively, then the value of its parameter p is **[1]**

a) $\frac{2}{3}$
b) $\frac{1}{3}$
c) $\frac{1}{4}$
d) $\frac{1}{2}$

7. In a series of three trials, the probability of two successes is 9 times the probability of three successes. Then, the probability of success in each trial is **[1]**

a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) $\frac{3}{4}$
d) $\frac{1}{3}$

8. The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is **[1]**

a) 3
b) 2
c) not defined
d) 1

9. In a kilometer race, A beats B by 50 meters or 10 seconds. The time taken by A to complete the race is: **[1]**

a) 200 seconds
b) 90 seconds
c) 120 seconds
d) 190 seconds

10. If A = $\begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$, B = $\begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}$, then $(A + B)^{-1}$ **[1]**

a) none of these
b) does not exist
c) is a skew-symmetric matrix
d) $A^{-1} + B^{-1}$

11. (18 $\times$ 10) (mod 7) is **[1]**

a) 4
b) 5
c) 3
d) 2

12. Given that x, y and b are real numbers and x < y, b > 0, then **[1]**

a) $\frac{x}{b} < \frac{y}{b}$
b) $\frac{x}{b} \geq \frac{y}{b}$
c) $\frac{x}{b} \leq \frac{y}{b}$
d) $\frac{x}{b} > \frac{y}{b}$

13. A man rows at a speed of 8 km/hr in still water to a certain distance upstream and back to the starting point in a river which flows at 4 km/hr. The average speed of the journey in km/hr, is **[1]**

a) 4
b) 6
c) 12
d) 8

14. Linear programming of linear functions deals with: **[1]**

a) Minimizing
b) Maximizing
c) Optimizing
d) Normalizing

15. The corner points of the feasible region determined by the following system of linear inequalities: **[1]**
2x + y $\leq$ 10, x + 3y $\leq$ 15, x, y $\geq$ 0 are (0, 0), (5, 0), (3, 4) and (0, 5).

Let Z = px + qy, where p, q > 0.

Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

a) p = 3q
b) q = 3p
c) p = 2q
d) p = q

16. A simple random sample consists of four observations 1, 3, 5, 7. What is the point estimate of population standard deviation? **[1]**

a) 2.58
b) 2.87
c) 3.1
d) 2.3

17. If the supply function for a commodity is p = $\sqrt{9+x}$ and the market price p_0 = 4, then producer's surplus is **[1]**

a) 10
b) $\frac{10}{3}$
c) 3
d) 15

18. Seasonal variations are **[1]**

a) Long term
b) Irregular
c) Short term
d) Sudden

19. **Assertion (A):** A 2 × 2 matrix A= [a_{ij}], whose elements are given by a_{ij} = i × j, is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$. **[1]**

Reason (R): If A is a 4 × 2 matrix, then the elements in A is 5.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. The function f be given by f(x) = $2x^3 - 6x^2 + 6x + 5$. **[1]**

Assertion (A): x = 1 is not a point of local maxima.

Reason (R): x = 1 is not a point of local minima.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the compound interest on ₹ 7000 at 6% p.a for 18 months compounded quarterly. [Use$(1.015)^6$ = 1.093] **[2]**

OR

Mr. Dinesh has two investment options either 10% per annum compounded semiannually or 9.8% per annum compounded quarterly. Which option is better for Mr. Dinesh? Given $(1.0245)^4$ - 1.1017.

22. The Production of cement by a firm in year 1 to 9 is given below: **[2]**

Year	1	2	3	4	5	6	7	8	9
Production in (Tonnes)	4	5	5	6	7	8	9	8	10

Calculate the trend values for the above series by the 3-yearly moving average method.

23. Evaluate the definite integral: **[2]**

$\int_{-1}^{2}$ f(x) dx, where f(x) = |x + 1| + |x| + |x - 1|

24. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then find the values of x, y, z and w. **[2]**

OR

Find the values of x and y, given that $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

25. In what ratio must a grocer mix two varieties of tea worth ₹ 60 per kg and ₹ 65 per kg so that by selling the mixture at ₹ 68.20 per kg may gain 10%? **[2]**

Section C

26. A machine costing ₹ 30,000 is expected to have a useful life of 4 years and a final scrap value of ₹ 4000. Find the annual depreciation charge using the straight-line method. Prepare the depreciation schedule. **[3]**

27. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes. **[3]**

OR

Solve the differential equation: $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$

28. A company suffers a loss of ₹1,000 if its product does not sell at all. Marginal revenue and Marginal cost functions for the product are given by MR = 50 - 4x and MC = -10 + x respectively. Determine the total profit function, break-even points and the profit maximization level of output **[3]**

29. From the following data calculate the 4-yearly moving averages and determine the trend values. **[3]**

Years	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Value	50.0	36.5	43.0	44.5	38.9	38.9	32.6	41.7	41.1	33.8

30. Ten cartons are taken at random from an automatic filling machine. The mean net weight of the cartons is 11.8 kg and the standard deviation 0.15 kg. Does the sample mean differ significantly from the intended weight of 12 kg? [Given that for d.f. = 9, $t_{0.05} = 2.26$] **[3]**

31. Let X denote the no of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form where K is some unknown constant **[3]**

$$P(\chi = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{, or } 2 \\ K(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

i. Find the value of K.

ii. What is the probability that you study at least two hours? Exactly two hours. At most two hours.

OR

A book of 525 pages contains 42 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages selected at random, will have at most 2 errors? (Given $e^{-0.8} = 0.45$)

Section D

32. An airline agrees to charter planes for a group. The group needs at least 160 first-class seats and at least 300 tourist class seats. The airline must use at least two of its model 314 planes which have 20 first-class and 30 tourist class seats. The airline will also use some of its model 535 planes which have 20 first-class seats and 60 tourist class seats. Each flight of a model 314 plane costs the company ₹ 100,000 and each flight of a model 535 plane costs ₹ 150,000. How many of each type of plane should be used to minimize the flight cost? Formulate this as an LPP. **[5]**

OR

Two tailors P and Q earn ₹ 150 and ₹ 200 per day respectively. P can stitch 6 shirts and 4 trousers a day, while Q can stitch 10 shirts and 4 trousers per day. How many days should each work to produce at least 60 shirts and 32 trousers at minimum labour cost?

33. A die is tossed twice. A success is getting an odd number on a random toss. Find the variance of the number of successes. **[5]**

OR

From a lot of 6 items containing 2 defective items, a sample of 4 items is drawn at random (without replacement). If the random variable X denotes the number of defective items in the sample, find:

i. the probability distribution of X.
ii. the mean of the distribution.
iii. the variance of the distribution.

34. Solve the system of inequations graphically: $2x + y \geq 8$, $x + 2y \geq 8$, $x + y \leq 6$ **[5]**

35. A loan of ₹ 400000 at the interest rate of 6.75 % p.a. compounded monthly is to be amortized by equal payments at the end of each month for 10 years. Find **[5]**

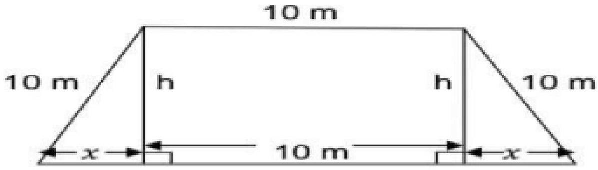
i. the size of each monthly payment.
ii. the principal outstanding at the beginning of 61st month.
iii. the interest paid in 61st payment.
iv. the principal contained in 61st payment.
v. total interest paid.

Given $(1.005625)^{120} = 1.9603$, $(1.005625)^{60} = 1.4001$)

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

The front gate of a building is in the shape of a trapezium as shown below. Its three sides other than base are 10m each. The height of the gate is h meter. On the basis of this information and figure given below answer the following questions:



i. How will you show the area A of the gate expressed as a function of x? (1)
ii. What is the value of $\frac{dA}{dx}$? (1)
iii. For which positive value of x, $\frac{dA}{dx} = 0$? (2)

OR

At the value of x where $\frac{dA}{dx} = 0$, area of trapezium is maximum then what is the maximum area of trapezium? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

What Is a Sinking Fund?

A sinking fund contains money set aside or saved to pay off a debt or bond. A company that issues debt will need to pay that debt off in the future, and the sinking fund helps to soften the hardship of a large outlay of revenue. A sinking fund allows companies that have floated debt in the form of bonds gradually save money and avoid a large lump-sum payment at maturity.

Example:

- Cost of Machine: ₹2,00,000/-

- Effective Life: 7 Years
- Scrap Value: ₹30,000/-
- Sinking Fund Earning Rate: 5%
- The Expected Cost of New Machine: ₹3,00,000/-

i. What is the money required for a new machine after 7 years? (1)
ii. What is the value of A, i and n here? (1)
iii. What formula will you use to get the requisite amount? (2)

OR

What amount should the company put into a sinking fund earning 5% per annum to replace the machine after its useful life? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Raja purchases 3 pens, 2 pencils and 1 mathematics instrument box and pays ₹41 to the shopkeeper. His friends, Daya and Anil purchases 2pens,1 pencil, 2 instrument boxes and 2 pens, 2 pencils and 2 mathematical instrument boxes respectively. Daya and Anil pays ₹ 29 and ₹ 44 respectively. Based on the above information answer the following:

i. What will be the cost of one pen? (1)
ii. What will be the cost of one pen and one pencil? (1)
iii. What will be the cost of one pen and one mathematical instrument box? (2)

OR

What will be the cost of one pencil and one mathematical instrumental box? (2)

Solution

Section A

1.
(d) 2B
Explanation:
2B

2. **(a)** Statistic
Explanation:
Statistic.

3. **(a)** ₹ 5000
Explanation:
Let sum invested be ₹ x, rate = 8%, time = 2 years
Amount = ₹ 5832
$\therefore$ 5832 = $x\left(1+\frac{8}{100}\right)^2$
$\Rightarrow$ 5832 - x $\times$ $\left(\frac{27}{25}\right)^2$
$\Rightarrow x=\frac{5832\times25\times25}{27\times27}$ = 5000
$\therefore$ Sum invested = ₹ 5000

4.
(d) 60
Explanation:
Here the objective function is given by:
F = 4x + 6y

Corner points	**Z = 4x +6 y**
(0, 2)	12 (Min.)
(3, 0)	12 (Min.)
(6, 0)	24
(6, 8)	72 (Max.)
(0, 5)	30

Maximum of F - Minimum of F = 72 - 12 = 60

5.
(d) Det (A)
Explanation:
$|B^{-1}AB| = |B^{-1}| \times |A| \times |B|$
$\frac{1}{B}\times |A| \times |B| = |A|$
$\frac{1}{B} \times |A| \times |B| = |A|$

6. **(a)** $\frac{2}{3}$
Explanation:
Given mean = np = 12 ...(i)
And we know that variance is square of standard deviation
so variance npq = 2^2 = 4 ...(ii)

Divide both the equation q = $\frac{1}{3}$

So p = 1 - q = $\frac{2}{3}$

7. **(a)** $\frac{1}{4}$

Explanation:

Given n = 3 and P(X = 2) = 9P(X = 3).

So, ${}^3C_2p^2 \cdot q = 9 \times {}^3C_3.p^3$

$\Rightarrow$ $3p^2q = 9p^3 \Rightarrow 3p^2(q - 3p) = 0$

$\Rightarrow$ q = 3p

$\because$ p + q = 1 $\Rightarrow$ p + 3p = 1 $\Rightarrow$ p $\frac{1}{4}$

8.

(c) not defined

Explanation:

As the term log $\left(\frac{d^2y}{dx^2}\right)$ is not a polynomial in $\frac{d^2y}{dx^2}$. So, the degree of the given differential equation is not defined.

9.

(d) 190 seconds

Explanation:

In a 1000 m race

50 m = 10 sec

1 m = $\frac{10}{50}$

1000 m of race will take = $\frac{10}{50} \times 1000$ = 200 sec

$\therefore$ Time taken by 'A' to complete 1000 m = (200 - 10) = 190 sec

10. **(a)** none of these

Explanation:

$A = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}$

A + B = $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

Here, we know that $(A + B)^{-1} \neq A^{-1} + B^{-1}$

$(A+B)^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = A + B$

11.

(b) 5

Explanation:

(18 $\times$ 10) (mod 7) = 18 (mod 7) $\times$ 10 (mod 7)

= 4 (mod 7) $\times$ 3 (mod 7)

= 12 (mod 7) = 5

12.

(d) $\frac{x}{b} > \frac{y}{b}$

Explanation:

x < y and b < 0

$\Rightarrow \frac{x}{b} > \frac{y}{b}$

13.

(b) 6

Explanation:

upstream speed = (8 - 4) km/hr = 4 km/hr

downstream speed = (8 + 4) km/hr = 12 km/hr

Let the distance covered be x km

$t_{upstream} = \frac{x}{4}$

4 $t_{upstream}$ = x

tup = $\frac{x}{4}$

$t_{downstream} = \frac{x}{12}$

$V_{avg} = \frac{\text{Total distance}}{\text{total times}}$

$= \frac{x+x}{t_{upstream}+t_{downstream}}$

$= \frac{2x}{t_{upstream}+t_{downstream}}$

$= \frac{2x}{\frac{x}{4}+\frac{x}{12}}$

$= \frac{2x}{x(\frac{x}{4}+\frac{x}{12})}$

$= \frac{2}{\frac{3+1}{12}} = \frac{2}{4} \times 12$ = 6 km/hr

Hence, Av speed of the journey is 6 km/hr.

14.

(c) Optimizing

Explanation:

Optimizing

15.

(b) q = 3p

Explanation:

Given the vertices of the feasible region are:

Q(0, 0)

A(5, 0)

B(3, 4)

C(0, 5)

Also given the objective function is Z = px + qy

Now substituting O, A, B and C in Z

Z at O(0, 0)	Z = P(0) + q(0) = 0
Z at A(5, 0)	Z = p(5) + q(0) = 5p + 0 = 5p
Z at B(3, 4)	Z = p(3) + q(4) = 3p + 4q
Z at C(0, 5)	Z = p(0) + q(5) = 0 + 5q

As per the condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5)

Then we can equate Z values at B and C, this gives

3p + 4q = 5q

3p = 5q - 4q

3p = q

16. **(a)** 2.58

Explanation:

2.58

17.

(b) $\frac{10}{3}$

Explanation:

Given P = $\frac{10}{x}$ and p_0 = 4

So, 4 = $\sqrt{9+x_0} \Rightarrow x_0 = 7$

P.S. $= 7 \times 4 - \int_0^7 \sqrt{9+x}dx = 28 - \left[\frac{2}{3}(9+x)^{\frac{3}{2}}\right]_0^7$

$= 28 - \left(\frac{128}{3} - \frac{54}{3}\right) = \frac{10}{3}$

18.

(c) Short term

Explanation:

Short term

19.

(c) A is true but R is false.

Explanation:

Assertion: In general, the matrix A of order 2 $\times$ 2 is given by A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Now, $a_{ij} = i \times j$, i = 1, 2 and j = 1, 2

$\therefore$ $a_{11} = 1$, $a_{12} = 2$, $a_{21} = 2$ and $a_{22} = 4$

Thus, matrix A is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

Reason: If A is a 4 $\times$ 2 matrix, then A has 4 $\times$ 2 = 8 elements.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

We have,

$f(x) = 2x^3 - 6x^2 + 6x + 5$

$\Rightarrow f'(x) = 6x^2 - 12x + 6 = 6(x - 1)^2$

and f''(x) = 12(x - 1)

Now, f'(x) = 0 gives x = 1.

Also, f''(1) = 0.

Therefore, the second derivative test fails in this case.

So, we shall go back to the first derivative test.

Using first derivatives test, we get x = 1 is neither a point of local maxima nor a point of local minima and so it is a point of inflexion.

Section B

21. P = ₹ 7000, r = 6% p.a. = 1.5% quarterly n = 18 months = 6 quarters

$\therefore$ C.I. = 7000 $\left[\left(1+\frac{1.5}{100}\right)^6 - 1\right]$

$= 7000[(1.015)^6 - 1]$

= 7000(1.093 - 1)

= 7000 $\times$ 0.093

= ₹ 651

OR

First option: Given r = 10% p.a.

p = 2 half years.

So, effective rate (per rupee) = $\left(1+\frac{10}{200}\right)^2$ - 1 = $(1.05)^2$ - 1

= 1.1025 - 1 = 0.1025 or 10.25%

Thus, effective rate = (0.1025) $\times$ 100% = 10.25%

Second option: Given r = 9.8 % p.a., p = 4 quarters

So, effective rate (per rupee) = $\left(1+\frac{9.8}{400}\right)^4$ - 1 = $(1.0245)^4$ - 1

= 1.1017 - 1 = 0.1017

Thus, effective rate = 0.1017 $\times$ 100% = 10.17%

Hence, the first option is better for Mr. Dinesh.

22. To calculate the trend values, we make the following table

Year	Production (in Tonnes)	Three yearly moving totals	Three yearly moving averages
1	4	-	-
2	5	14	4.67
3	5	16	5.33
4	6	18	6
5	7	21	7
6	8	24	8
7	9	25	8.33
8	8	27	9
9	10	-	-

23. When -1 $\leq$ x $\leq$ 0, f(x) = (x + 1) - x - (x - 1) = 2 - x;

when 0 $\leq$ x $\leq$ 1, f(x) = (x + 1) + x - (x - 1) = x + 2;

when 1 $\leq$ x $\leq$ 2, f(x) = (x + 1) + x + (x - 1) = 3x.

$\therefore \int\limits_{-1}^{2} f(x)dx = \int\limits_{-1}^{0} f(x)dx + \int\limits_{0}^{1} f(x)dx + \int\limits_{1}^{2} f(x)dx$

= $\int\limits_{-1}^{0}(2-x)dx + \int\limits_{0}^{1}(x+2)dx + \int\limits_{1}^{2} 3xdx$

= $\left[2x - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} + 2x\right]_{0}^{1} + \left[\frac{3x^2}{2}\right]_{1}^{2}$

= (0 - 0) $- \left(-2 - \frac{1}{2}\right) + \left(\frac{1}{2} + 2\right)$ - (0 + 0) $+ \left(6 - \frac{3}{2}\right)$

= $\frac{5}{2} + \frac{5}{2} + \frac{9}{2} = \frac{19}{2}$

24. We have, $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

By equality of matrix, x + y = 6 and xy = 8

$\Rightarrow$ x = 6 - y and (6 - y)y = 8

$\Rightarrow$ y^2 - 6y + 8 = 0

$\Rightarrow$ y^2 - 4y - 2y + 8 = 0

$\Rightarrow$ (y - 2)(y - 4) = 0

$\Rightarrow$ y = 2 or y = 4

$\therefore$ x = 6 - 4 = 2

or x = 6 - 4 = 2 [$\because$ x = 6 - y]

Also, z + 6=0

$\Rightarrow$ z = -6 and w = 4

$\therefore$ x = 2, y = 4 or x = 4, y = 2, z = -6 and w = 4

OR

Given $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x.1 + y.2 \\ 3y.1 + x.2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\Rightarrow$ x + 2y = 3, 2x + 3y = 5 $\Rightarrow$ x = 1, y = 1.

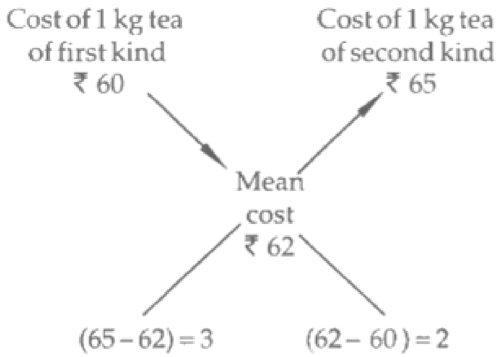
25. We have,

S.P. of mixture = ₹ 68.20 and Gain = 10%

$\therefore$ C.P. = $\frac{SP}{1+\frac{Gain}{100}}$

$\Rightarrow$ C.P. = $\left(\frac{68.20}{1+\frac{10}{100}}\right)$ = $\left(\frac{682}{11}\right)$ = ₹ 62

The allegation grid is as given below:



By using the rule of allegation, we obtain

Tea of first kind: Tea of second kind = 3 : 2

Hence, the grocer must mix in the ratio 3 : 2

Section C

26. We are given that

C = 30,000; n = 4; S = 4000

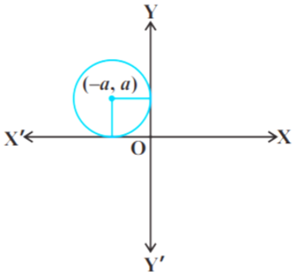
Annual depreciation = $\frac{C-S}{n}$

$= \frac{30000-4000}{4}$

= 6500

Depreciation schedule

Year	Annual depreciation	Accumulated depreciation	Book Value
0	0	0	30,000
1	6500	6500	23,500
2	6500	13000	17,000
3	6500	19,500	10,500
4	6500	26,000	4000

27. 

Eq. of circle is

$(x + a)^2 + (y - a)^2 = a^2$ ...(1)

$\Rightarrow x^2 + y^2 + 2ax - 2ay + a^2 = 0$

Differentiating both sides w.r.t x, we get,

$2x + 2yy' + 2a - 2ay' = 0$

$\Rightarrow x + yy' = a(y' - 1)$

$\Rightarrow \frac{x+yy'}{y'-1} = a$

Put the value of a in eq (1), we get,

$(x + \frac{x+yy'}{y'-1})^2 + (y - \frac{x+yy'}{y'-1})^2 = (\frac{x+yy'}{y'-1})^2$

$\Rightarrow \left(\frac{x(y'-1)+x+yy'}{y'-1}\right)^2 + \left(\frac{y(y'-1)-x-yy'}{y'-1}\right)^2 = \left(\frac{x+yy'}{y'-1}\right)^2$

$\Rightarrow \left(\frac{xy'-x+x+yy'}{y'-1}\right)^2 + \left(\frac{-x-y}{y'-1}\right)^2 = \left(\frac{x+yy'}{y'-1}\right)^2$

$\Rightarrow \left(\frac{(x+y)y'}{y'-1}\right)^2 + \left(\frac{-(x+y)}{y'-1}\right)^2 = \left(\frac{x+yy'}{y'-1}\right)^2$

$\Rightarrow y'^2(x + y)^2 + (x + y)^2 = (x + yy')^2$

$\Rightarrow (x + y)^2(y'^2 + 1) = (x + yy')^2$

OR

The given differential equation is

x log x$\frac{dy}{dx}$ + y = $\frac{2}{x}$ log x

$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x^2}$ ...(i)

This is a linear differential equation of the form

$\frac{dy}{dx}$ + Py = Q, where P = $\frac{1}{x\log x}$ and Q = $\frac{2}{x^2}$

$\therefore$ I.F. = $e^{\int Pdx} = e^{\int \frac{1}{x\log x}dx} = e^{\int \frac{1}{t}dt}$, where t = log x

$\Rightarrow$ I.F. = $e^{\log t}$ = t = log x

Multiplying both sides of (i) by I.F. = log x, we get

log x$\frac{dy}{dx} + \frac{1}{x}$y = $\frac{2}{x^2}$ log x

Integrating both sides with respect to x, we get

y log x = $\int \frac{2}{x^2}$ log x dx + C [Using: y(I.F.) = $\int$ Q (I.F.) dx + c]

$\Rightarrow$ y log x = $2\int \underset{I}{\log x}\, \underset{II}{x^{-2}}dx + C$

$\Rightarrow$ y log x = $2\left\{\log x\left(\frac{x^{-1}}{-1}\right) - \int \frac{1}{x}\left(\frac{x^{-1}}{-1}\right)dx\right\}$ + C

$\Rightarrow$ y log x = $2\left\{-\frac{\log x}{x} + \int x^{-2}dx\right\}$ + C

$\Rightarrow$ y log x = $2\left\{-\frac{\log x}{x} - \frac{1}{x}\right\}$ + C

$\Rightarrow$ y log x = $-\frac{2}{x}$(1 + log x) + C, which gives the required solution.

28. Let P denote the profit function. Then,

$\frac{dP}{dx}$ = MR - MC

$\Rightarrow \frac{dP}{dx}$ = (50 - 4x) - (-10 + x)

$\Rightarrow \frac{dP}{dx}$ = 60 - 5x and $\frac{d^2P}{dx^2}$ = -5

For maximum value of P, we must have

$\frac{dP}{dx}$ = 0 $\Rightarrow$ 60 - 5x = 0 $\Rightarrow$ x = 12

Clearly, $\frac{d^2P}{dx^2}$ = -5 < 0 for all x.

So, profit P is maximum when 12 units are produced. Thus, the profit maximization level of output is 12 units.

Now, $\frac{dP}{dx}$ = 60 - 5x

$\Rightarrow$ P = $\int(60 - 5x)dx + k$ ... [On intergrating]

$\Rightarrow$ P = 60x - $\frac{5}{2}x^2$ + k ... (i)

where k is the constant of integration

It is given that the company suffers a loss of ₹ 1000, if its product does not sell at all i.e. P = -1000 at x = 0. Substituting these values in (i), we obtain k = -1000.

Putting k = -1000 in (i), we obtain:

P = 60x - $\frac{5}{2}x^2$ + 1000

This is the total profit function. For break-even points

P = 0 $\Rightarrow$ 60x - $\frac{5}{2}x^2$ + 1000 = 0 $\Rightarrow$ 5x^2 - 120x + 2000 = 0

$\Rightarrow$ x^2 - 24x + 400 = 0

This equation does not give real values of x. So, there is no break-even point.

29. Calculation of Trend values by it four yearly Moving Averages:

Year	**Value**	**4-yearly centered Moving Total**	**4-yearly Moving Average (Trend values)**	**4-yearly centered Moving Average**
2012	50.0	-		
2013	36.5	-		
		174.0	43.5	
2014	43.0	-		42.12
		162.9	40.73	

2015	44.5	-		41.03
		165.3	41.33	
2016	38.9	-		40.03
		154.9	38.73	
2017	38.9	-		38.38
		152.1	38.03	
2018	32.6	-		38.31
		154.3	38.58	
2019	41.7	-		37.94
		149.2	37.3	
2020	41.1	-		
2021	33.8	-		

30. $\mu =$ Population mean = 12 Kg

$\overline{X}$ = Sample mean = 11.8 Kg

n = 10

Sample standard deviation = s = 0.15

Null Hypothesis H_0 = There is no significance between the sample mean $\overline{X}$ and the population mean μ.

Alternate Hypothesis H_1 = There is significance between the sample mean $\overline{X}$ and the population mean μ

Let t be the test statistic given by

$$t = \frac{\overline{X}-\mu}{\frac{s}{\sqrt{n-1}}}$$

$$t = \left(\frac{11.8-12}{0.15}\right) \times 3$$

= -4

The test statistic t follows student t-distribution with (10-1)=9 degrees of freedom

It is given that $t_{0.05}$ = 2.26

We observe that,

|t| = 4>2.26

$\implies$ Calculate |t| > tabulated $t_9(0.05)$

So, the null hypothesis is rejected at a 5% level of significance.

Hence there is a significance between the sample mean $\overline{X}$ and the population mean μ.

31. The probability distribution of x is

X	0	1	2	3	4
P(X)	0.1	K	2K	2K	K

i. $\sum_{i=1}^{n} pi = 1$

0.1 + K + 2K + 2K + K = 1

K = 0.15

ii. p (study atleast two hours) = p (x ≥ 2)

= 2K + 2K + K

= 5K

$= 5 \times 0.15$

= 0.75

p (Study exactly two hours) = p(x = 2)

= 2K

$= 2 \times 0.15$

= 0.3

p (Study at most two hours)= p(x$\leq$2)

= p(x=0) + p(x=1) + p(x=2)

= 0.1+ k+2k

= 0.1+3k = 0.1+3(0.15)

= 0.1+0.45 = 0.55

OR

Here n = 10 $p=\frac{42}{525}=0.08$

mean = m = np = 10 $\times$ 0.08 = 0.8

Then poisson distribution is given by:

$p(r)=\frac{e^{-m}m^r}{r!}=\frac{e^{-0.8}\times(0.8)^r}{r!}$

Here, we have atmast 2 error

i.e. r = (0, 1, 2)

$\therefore$ P(0) $=\frac{e^{-0.8}\times(0.8)^0}{0!}=\frac{0.45\times1}{1}=0.45$

P(1) $=\frac{e^{-0.8}\times(0.8)^1}{1!}=\frac{0.45\times0.8}{1}=0.36$

P(2) $=\frac{e^{-0.8}\times(0.8)^2}{2!}=\frac{0.45\times0.84}{2}=0.144$

Section D

32.

Model 314				
Variable	X		Y	
F Class	20x	+	20y	$\geq$ 160
T Class	30x	+	60y	$\geq$ 300
Cost	1.x lakh	+	1.5y lakh	Z

The above LPP can be presented in the table above.

The flight cost is to be minimized i.e; Min Z = x + 1.5y the constraints

x $\geq$ 2 at least 2 planes of model 314 must be used

y $\geq$ 0 at least 1 plane of model 535 must be used

20x + 20y $\geq$ 160 require at least 160 F class seats

30x + 60y $\geq$ 300 require at least 300 T class seats

Solving the above inequalities as equations we get,

When x = 0, y = 8 and when y = 0, x = 8

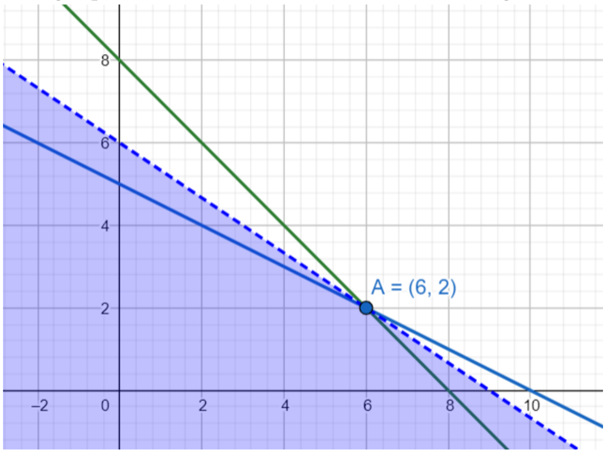
We get an unbounded region 8 - E - 10 as a feasible solution. Plotting the corner points and evaluating we have,

Corner point	**Value of Z = x + 1.5y**
10, 0	10
0, 8	12
6, 2	9

Since we obtained an unbounded region as the feasible solution a plot of Z(x + 1.5 y < 9) is plotted.

Since there are no common points point E is the point that gives a minimum value.

Using 6 planes of model 314 & 2 of model 535 gives minimum cost of 9 lakh rupees.



OR

Let the tailor P work for x days and the tailor Q work for y days respectively.

Here, the problem can be formulated as an L.P.P. as follows:

Minimize Z = 150x + 200y

Subject to the constraints:

$6x + 10y \geq 60$

or $3x + 5y \geq 30$ ...(i)

$4x + 4y \geq 32$

or $x + y \geq 8$ ...(ii)

and $x \geq 0$, $y \geq 0$

Converting them into equations we obtain the following equations:

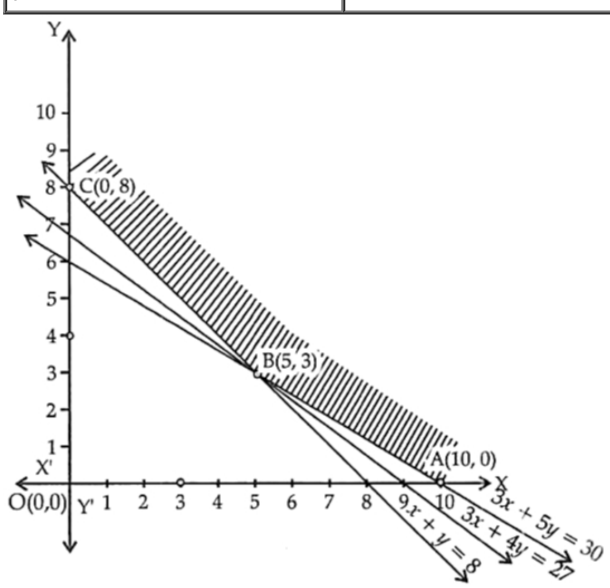
3x + 5y = 30, x + y = 8

$\Rightarrow y = \frac{30-3x}{5}$

$\Rightarrow y = 8 - x$

x	0	10	5
y	6	0	3

x	0	8	5
y	8	0	3



The shaded region in the diagram represent the feasible region.

The corner points are A(10, 0), B(5, 3) and C(0, 8)

At the corner point the value of Z = 150x + 200y

At (10, 0) Z = 1500

At B(5, 3) $Z = 150 \times 5 + 200 \times 3$

= 750 + 600 = 1350

At C(0, 8) Z = 1600

As the feasible region is unbounded, we draw the graph of the half-plane.

150x + 200y < 1350

3x + 4y < 27

There is no point common with the feasible region, therefore, Z has minimum value.

Minimum value of Z is ₹ 1350 and it occurs at the point B(5, 3).

Hence, the labour cost is ₹ 1350 when P works for 5 days and Q works for 3 days.

33. Let X be a random variable denoting the number of successes in two tosses of a die. Then, X can take values 0, 1, 2.

Let S_i and F_i denote the success and failure respectively in i^{th} toss. Then,we have,

$P(S_i)$ = Probability of getting an odd number in i^{th} toss = $\frac{3}{6} = \frac{1}{2}$

and $P(F_i)$ = Probability of not getting an odd number in i^{th} toss = $\left(1 - \frac{1}{2}\right) = \frac{1}{2}$

Now, P (X = 0) = Probability of getting no success in two tosses of a die

$\Rightarrow$ P(X = 0) = $P(F_1 \cap F_2)$

$\Rightarrow$ P(X = 0) = $P(F_1)\, P(F_2)$ [by Multiplication Theorem]

$\Rightarrow$ P(X = 0) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ [$\because P(F_1) = P(F_2) = \frac{1}{2}$]

P(X = 1) = Probability of getting one success in two tosses of a die

$\Rightarrow$ P(X = 1) = $P\left((S_1 \cap F_2) \cup (F_1 \cap S_2)\right)$

$\Rightarrow$ P(X = 1) = $P\left(S_1 \cap F_2\right) + P\left(F_1 \cap S_2\right)$ = $P(S_1)\, P(F_2) + P(F_1)\, P(S_2)$ = $\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

and, P (X = 2) = Probability of getting two successes in two tosses of a die

$\Rightarrow$ P (X = 2) = $P\left(S_1 \cap S_2\right)$ = $P(S_1)\, P(S_2)$ = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Therefore, the probability distribution of X i.e. the number of successes in two tosses of a die is as follows:

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Computation of variance:

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{4}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{4}$	$\frac{1}{2}$	1
		$\Sigma p_i x_i = 1$	$\Sigma p_i x_i^2 = \frac{3}{2}$

Therefore, we have $\Sigma p_i x_i = 1$ and $\Sigma p_i x_i^2 = \frac{3}{2}$

$\therefore$ Var(X) = $\Sigma p_i x_i^2 - \left(\Sigma p_i x_i\right)^2$ = $\frac{3}{2} - 1 = \frac{1}{2}$

OR

Total number of items = 6, number of defective items = 2,

4 items are drawn at random.

Given X denote the number of defective items drawn, then X can take values 0, 1, 2.

i. P(X = 0) = P(no defective item) $= \frac{{}^4C_4}{{}^6C_4} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3} = \frac{1}{15}$,

P(X = 1) = P(one defective item) $= \frac{{}^4C_3 \times {}^2C_1}{{}^6C_4} = \frac{8}{15}$,

P(X = 2) = P(two defective items) $= \frac{{}^4C_2 \times {}^2C_2}{{}^6C_4} = \frac{6}{15}$.

$\therefore$ Probability distribution of number of defective items drawn is $\begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{15} & \frac{8}{15} & \frac{6}{15} \end{pmatrix}$.

ii. we construct the following table:

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{15}$	0	0
1	$\frac{8}{15}$	$\frac{8}{15}$	$\frac{8}{15}$

2	$\frac{6}{15}$	$\frac{12}{15}$	$\frac{24}{15}$
Total		$\frac{20}{15}$	$\frac{32}{15}$

Mean $= \Sigma p_i x_i = \frac{20}{15} = \frac{4}{3}$. .

iii. Variance $= \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{32}{15} - \frac{16}{9} = \frac{16}{45}$.

34. First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality. You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y-intercepts always.

$2x + y \geq 8$

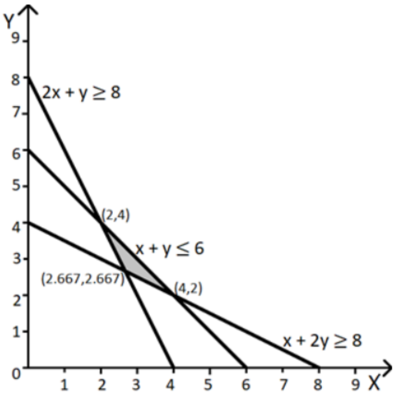
x	0	2	4
y	8	4	0

$x + 2y \geq 8$

x	0	4	8
y	4	2	0

$x + y \leq 6$

x	0	3	6
y	6	3	0



35. i. Given P = ₹ 400000, n = 120, i = $\frac{6.75}{1200}$ = 0.005625

$\therefore$ EMI = $\frac{400000 \times 0.005625 \times (1.005625)^{120}}{(1.005625)^{120} - 1}$

$= \frac{400000 \times 0.005625 \times 1.9603}{0.9603}$ = ₹4593.

ii. Principal outstanding at the beginning of 61 months

$= \frac{\text{EMI}\left[(1+i)^{n-k+1} - 1\right]}{i(1+i)^{n-k+1}} = \frac{4593\left[(1.005625)^{120-61+1} - 1\right]}{0.005625(1.005625)^{120-61+1}}$

$= \frac{4593(1.4001-1)}{0.005625 \times 1.4001}$ = ₹ 233336.89

iii. Interest paid in 61st payment = $\frac{\text{EMI}\left[(1+i)^{n-k+1} - 1\right]}{(1+i)^{n-k+1}}$

$= \frac{4593 \times 0.4001}{1.4001}$ = ₹1312.52

iv. Principal paid in 61st payment = EMI - Interest paid in 61st period

= ₹ 4593 - ₹ 1312.52 = ₹ 3280.48

v. Total interest paid = n $\times$ EMI - P

= 120 $\times$ 4593 - 400000 = ₹ 151160.

Section E

36. i. $(10 + x)\sqrt{100 - x^2}$

ii. $\frac{-2x^2 - 10x + 100}{\sqrt{100 - x^2}}$

iii. 5

OR

$75\sqrt{3}$ sq.m

37. i. Cost of new machine = ₹ 300000

Scrap value of old machine = ₹ 30000

Hence, the money required for new machine after 7 years

= ₹ 300000 - ₹ 30000 = ₹ 270000

ii. A = ₹ 270000, i = $\frac{5}{100}$ = 0.05, n = 7

iii. A = $R\left[\frac{(1+i)^n-1}{i}\right]$

OR

Cost of new machine = ₹300000

Scrap value of old machine = ₹30000

Hence, the money required for new machine after 7 years

= ₹300000 - ₹30000 = ₹270000

So, we have A = ₹270000, i = $\frac{5}{100}$ = 0.05, n = 7

Using formula, A = $R\left[\frac{(1+i)^n-1}{i}\right]$, we get

270000 = R $\left[\frac{(1.05)^7-1}{0.05}\right]$

[Let x = $(1.05)^7$

$\Rightarrow$ log x = 7 log 1.05 = 7 $\times$ 0.0212 = 0.1484

$\Rightarrow$ x = antilog 0.1484

$\Rightarrow$ x = 1.407

$\Rightarrow R = \frac{270000 \times 0.05}{(1.05)^7-1}$

$\Rightarrow R = \frac{13500}{1.407-1} = \frac{13500}{0.407}$

$\Rightarrow$ R = 33169.53

Hence, the company should deposit ₹33169.53 at the end of each year for 7 years.

38. i. ₹ 2

ii. ₹ 17

iii. ₹ 7

OR

₹ 20